V_1: Unifying Generation and Self-Verification for Parallel Reasoners

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Test-time scaling for complex reasoning tasks shows that leveraging inference-time compute, by methods such as independently sampling and aggregating multiple solutions, results in significantly better task outcomes. However, a critical bottleneck is *verification*: sampling is only effective if correct solutions can be reliably identified among candidates. While existing approaches typically evaluate candidates independently via scalar scoring, we demonstrate that models are substantially stronger at **pairwise self-verification**. Leveraging this insight, we introduce V_1, a framework that unifies generation and verification through efficient pairwise ranking. V_1 comprises two components: V_1-**Infer**, an uncertainty-guided algorithm using a tournament-based ranking that dynamically allocates self-verification compute to candidate pairs whose relative correctness is most uncertain; and V_1-**PairRL**, an RL framework that **jointly trains** a single model as both generator and pairwise self-verifier, ensuring the verifier adapts to the generator's evolving distribution. On code generation (LiveCodeBench, CodeContests, SWE-Bench) and math reasoning (AIME, HMMT) benchmarks, V_1-Infer improves Pass@1 by up to 10% over pointwise verification and outperforms recent test-time scaling methods while being significantly more efficient. Furthermore, V_1-PairRL achieves 7–9% test-time scaling gains over standard RL and pointwise joint training, and improves base Pass@1 by up to 8.7% over standard RL in a code-generation setting.

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1 Introduction

Large language models (LLMs) have demonstrated remarkable problem-solving abilities, largely driven by the paradigm of "System 2" thinking [6, 27, 28] of generating extended chains of thought to reflect, refine, and verify answers at inference time. *Parallel reasoning*, which complements this sequential "deep thinking" by sampling multiple independent chains of thought to explore diverse solution paths, has emerged as a powerful technique for test-time scaling [4, 17, 29, 38, 44]. In this setup, parallel sampling of multiple chains-of-thought is followed by an aggregation step to select the final answer. The simplest form of aggregation is to select the most common solution among the set of candidates (majority voting). While this suffices for domains like math where answers are objective and easily verifiable [44], this can not be used for more general domains which do not admit objective ground truth answers. Instead, the ability to accurately *self-verify* solutions can support an aggregation method that selects the correct solution from the candidate set, as long as it exists in the set, even if it isn't the most common solution. Thus, taking full advantage of parallel reasoning fundamentally hinges on *accurate self-verification*: sampling N solutions is useful if the

model can reliably identify the correct one.

Our experiments identify a critical bottleneck in existing approaches that use models' intrinsic verification capabilities to take advantage of inference-time compute: without a globally comparable scale of solution quality, existing models are not calibrated to evaluate candidate solutions independent of one another. In addition, existing work suggests that in such settings, models used as verifiers are biased towards positively evaluating their own samples, even if those samples are incorrect [21]. We find that, instead, self-verifying between candidate solutions via pairwise comparison leads to more robust and accurate outcomes. We explore two core questions. First, *how can we enable LLMs to more accurately self-verify candidate solutions obtained via parallel reasoning, by leveraging pairwise candidate self-verification?* Second, given that self-verification is typically applied after a model has already been trained, *can we instead train models to be better at self-verification for parallel reasoning?*

While pairwise ranking has been extensively studied in reward modeling for LLM alignment [3, 58], this approach remains underexplored for self-verification in a parallel reasoning setting. Additionally, while reinforcement learning is commonly used to improve the solution-generation capabilities of LLMs on verifiable domains such as code and math [6, 34], no existing methods effectively utilizes the parallel reasoning chains of LLMs at training time to jointly optimize both the generation and self-verification capabilities, resulting in distribution shifts at inference time. We demonstrate that inducing pairwise self-verification capabilities during RL training is an effective technique to improve test-time scaling performance for parallel reasoners. We leverage our observations to develop $\mathbf{V}_1$, a unified framework that includes a strong inference time scaling algorithm for parallel reasoning, as well as an reinforcement learning framework that induces self-verification capabilities during RL training of LLMs with verifiable rewards.

Our contributions include the following:

1. We show that in parallelized reasoning, independent self-verification of candidate solutions suffers from calibration collapse due to lack of comparative reference. On the other hand, self-aggregation methods like recursive self-aggregation [RSA; 43] induce diversity collapse where Pass@N monotonically decreases with aggregation steps. This motivates pairwise verification as a principled alternative for self-verification and an orthogonal methodology for test-time scaling without loss of diversity.

2. We develop $\mathbf{V}_1$-Infer, an uncertainty-guided pairwise verification algorithm. Rather than scoring solutions in isolation, it pairs candidates by employing a Swiss-system tournament refinement strategy that dynamically allocates verification compute to the most uncertain pairs. This approach provides a significant boost in selection accuracy, effectively improving the performance of the model closer to Pass@N of the original sampled responses. Notably, we find that $\mathbf{V}_1$-Infer outperforms or matches RSA on tasks where aggregation leads to diversity collapse, while requiring significantly fewer verification calls.

3. We develop $\mathbf{V}_1$-PairRL, an RL framework that co-trains a single model as both generator and pairwise self-verifier. Unlike prior co-training approaches that rely on pointwise rewards [18, 32] or offline data [43], $\mathbf{V}_1$-PairRL uses an online, co-evolving objective where generation and pairwise verification improve together. This ensures that as the generator improves, the verifier trains on in-distribution data from the model's current capabilities, thereby leading to stronger self-verification capabilities at inference time.

We evaluate our framework on code generation (LiveCodeBench, CodeContests) and math reasoning (AIME, HMMT) benchmarks. $\mathbf{V}_1$-Infer improves Pass@1 by up to 10% over pointwise verification and matches or exceeds RSA with a fraction of the compute. $\mathbf{V}_1$-PairRL achieves 7–9% test-time scaling gains over pointwise co-training standard RL, and improves base Pass@1 by up to 8.7% over the standard RL. Our results demonstrate that unified training for solution generation and self-verification capability, coupled with our pairwise self-verification technique, enables effective parallel reasoning.

2 Related Work

Parallel reasoning and test-time scaling. Test-time scaling typically follows two paradigms: sequential refinement [23] or parallel generation of multiple reasoning paths [4, 29, 33]. While parallel scaling allows

for the exploration of diverse solutions, it necessitates a robust mechanism to verify and select the correct output. Existing approaches often rely on access to ground-truth verification signals during inference. For instance, mathematical reasoning often leverages majority voting based on exact answer matching [38, 44], while code generation methods rely on executable test cases or execution feedback [8, 14]. In contrast, we focus on *self-verification*, where the model must judge the quality of its own parallel generations without access to external feedback or ground-truth oracles.

Self-verification and self-aggregation. Early work demonstrated that LLMs can verify their own Chain-of-Thought (CoT) reasoning [46], though recent studies indicate that pointwise self-verification suffers from a bias toward accepting incorrect solutions [21]. While sequential self-refinement [23, 39] explores verification, it does not address the parallel reasoning setting. Alternatively, self-aggregation methods combine solutions from the same model [24, 43] but often suffer from diversity collapse, leading to the loss of correct solutions. Our work addresses these limitations by adopting pairwise self-verification, which mitigates pointwise bias and preserves solution diversity better than aggregation-based approaches.

Generative verifiers and Co-training. Generative reward models, which produce reasoning before scoring, have been shown to outperform discriminative approaches [25, 53], with pairwise ranking often proving more effective than absolute scoring [10, 42]. In this context, Zhao et al. [55] propose AggLM, which explicitly trains an aggregator via RL to synthesize correct answers for improving majority voting. However, these approaches typically rely on *separate* verifier or aggregator models, incurring significant overhead in compute, memory, and data curation. Recent co-training methods attempt to unify generation and verification to mitigate this cost [18, 32, 45], but they predominantly rely on pointwise verification rewards. Our V_1-PairRL framework advances this by co-training a single model for *pairwise* self-verification, eliminating the need for external verifiers while enabling efficient online learning.

See Appendix §A for more discussion about prior work and extension to this section.

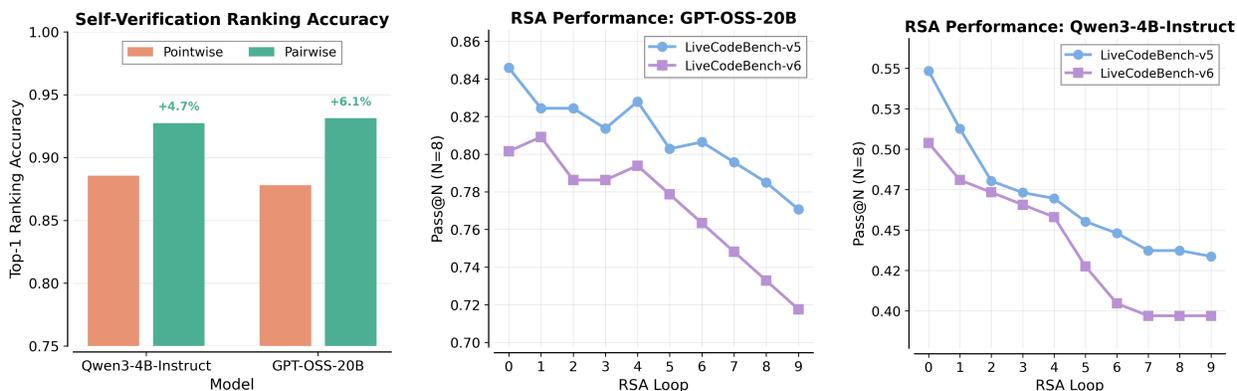# 3 Limitations of Current Self-Verification and Aggregation Approaches

Figure 1: (**Left**) Pairwise self-verification (using V_1-Infer, §4) outperforms pointwise self-verification in self-verification measured on problems which have both correct and incorrect solutions in their parallel generations (Results with GPT-OSS-20B on LiveCodeBench-V6 prompts). (**Middle and Right**) Recursive self-aggregation on LiveCodeBench benchmarks shows declining Pass@N (diversity collapse) for both GPT-OSS-20B and Qwen3-4B-Instruct. See §3 for more details.

From a test-time scaling perspective without external verifiers, parallel reasoning offers two prevalent mechanisms to generate the final solution: **a) self-selection** (verifying and choosing the best candidate) and
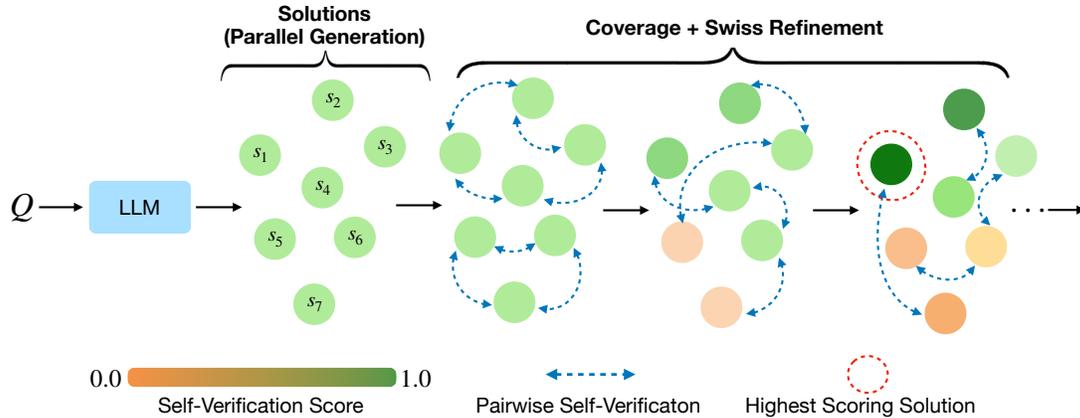
Figure 2: Swiss Refinement Overview. Increasing pairwise verifications enables LLMs to better self-verify for selecting the best response among N self-generated solutions. See Section 4

b) self-aggregation (combining solutions to generate a better one). We analyze the limitations of current approaches in both categories to motivate our pairwise framework [1].

Pointwise self-verification suffers from calibration collapse. Standard pointwise verification assigns scalar scores to solutions in isolation. This approach is fundamentally limited by the lack of a comparative reference set. Statistically, latent utilities in choice models (e.g., Bradley–Terry) are identifiable only up to monotonic transformations, meaning absolute scores lack a globally comparable scale [2]. Consequently, pointwise scores exhibit high variance and poor cross-context calibration, often over-scoring plausible but incorrect solutions. Christiano et al. [3] noted that for learning from human preferences, relative scores are much easier to provide for humans compared to absolute scores. *Pairwise* judgments simplify the task to a well-posed relative comparison. As shown in Figure 1 (left), this shift to relative ranking yields significantly higher top-1 self-ranking accuracy.

Self-aggregation leads to reduction in pass@N and diversity collapse. Self-aggregation-based methods prompt the same LLM to consolidate parallelly generated solutions into one solution. While self-aggregation methods [12, 16, 24, 43] can help consolidate parallel reasoning chains and improve Pass@1, they may lead to *diversity collapse*. As shown in Figure 1 (Middle and Right), the Pass@N score, representing the probability that *at least one* correct solution exists in a set of generated N solutions, monotonically decreases as aggregation steps increase for RSA. This indicates that RSA frequently discards or degrades correct outlier solutions during refinement. Specifically, since the refined Pass@1 rarely exceeds the *initial* Pass@N of the raw samples, the value of self-aggregation is unclear compared with a strong self-verifier that can select the best answer and reach near Pass@N performance. Instead of relying on implicit self-verification capabilities of aggregation based method, explicit and accurate self-verification can provide orthogonal improvements to aggregator-based approaches since each aggregation step (that may induce diversity collapse) can benefit from self-verification (that maintains pass@N), which can help select promising candidate solutions to aggregate.

> **Takeaway: Key Limitations of Existing Self-Verification and Aggregation Approaches**
>
> Pointwise self-verification lacks calibration due to the absence of comparative references, while self-aggregation methods suffer from diversity collapse that discards correct solutions during aggregation. Pairwise self-verification offers a principled orthogonal method that is both better calibrated and diversity-preserving.

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[1] Majority voting is less general and only applicable to scenarios with objective ground-truth answers, such as math.

4 Improving the Self-Verification Capability of LLMs using V_1-Infer

Pointwise verification is limited in its ability to capture relative nuances and lacks calibration. These limitations, coupled with a desire to maintain solution diversity, motivate us to propose V_1-*Infer*, a pairwise self-verification framework designed for parallel reasoning. While we can use pairwise self-verification to score responses by the model, naively pairing solutions may require quadratic number of $C(N, 2)$ self-verification attempts. Our approach, detailed in Algorithm 1, consists of a weighted aggregation mechanism and a two-phase budgeting strategy designed to maximize information gain with each new pair which allows us to efficiently scale test-time self-verification compute while achieving significant improvement in reasoning performance. A high-level overview of our approach is presented in Fig. 2
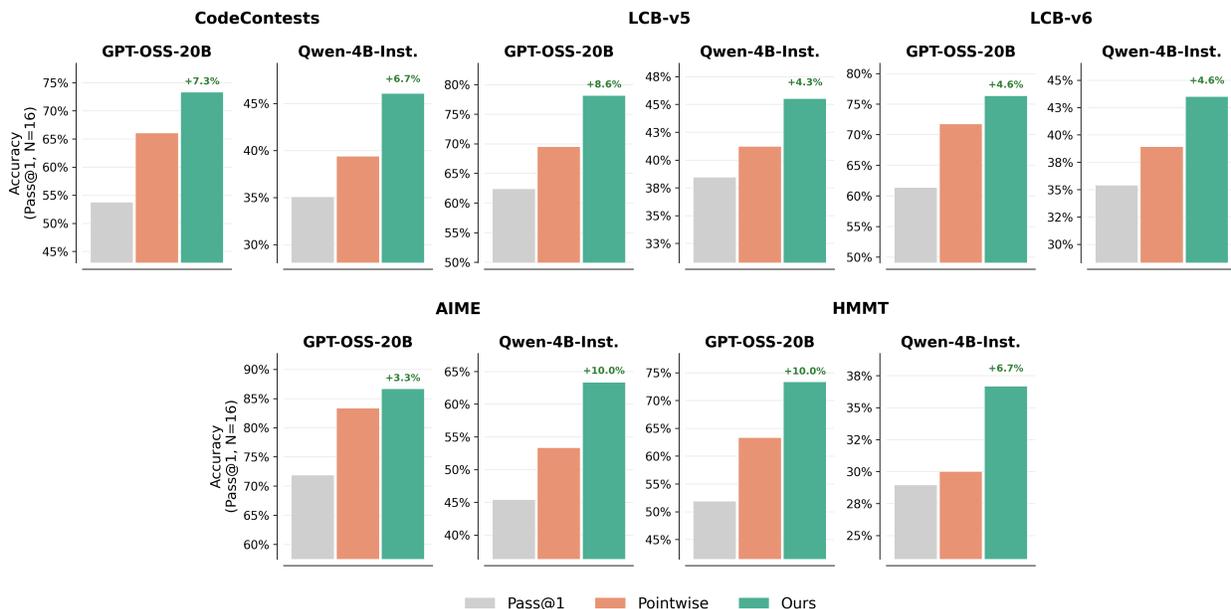

Figure 3: Performance after self-verification using V_1-Infer compared with pointwise self-verification across benchmarks and models at N=16 base generations. Results presented for GPT-OSS-20B and Qwen-4B-Instruct-2507. Results for GPT-OSS-120B and Qwen-4B-Thinking-2507 are in Fig. 11 and show similar trends.
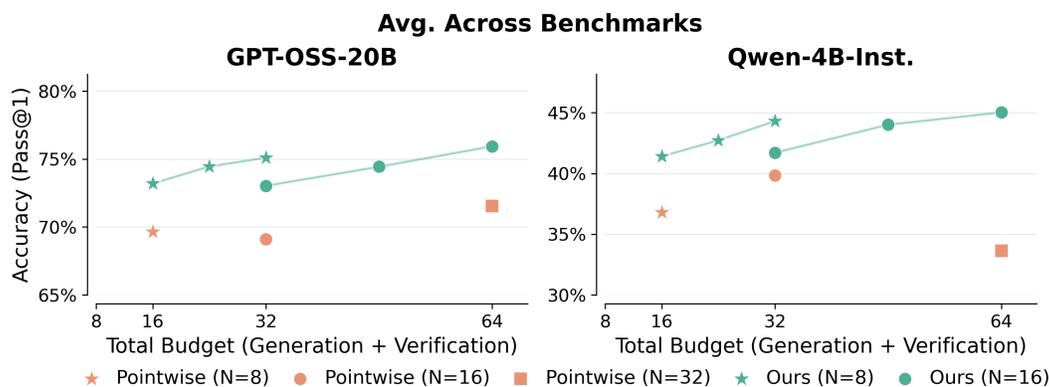

Figure 4: Accuracy vs. total budget (generation + verification calls). Stars, circles, and squares denote $N = 8$, $N = 16$, and $N = 32$ base generations from the LLM, while the total budget $= N + V$ where V is the number of verification calls. V_1-Infer consistently outperforms pointwise self-verification at equivalent budgets and shows monotonic performance scaling with compute. See Fig. 14 for per-benchmark results.
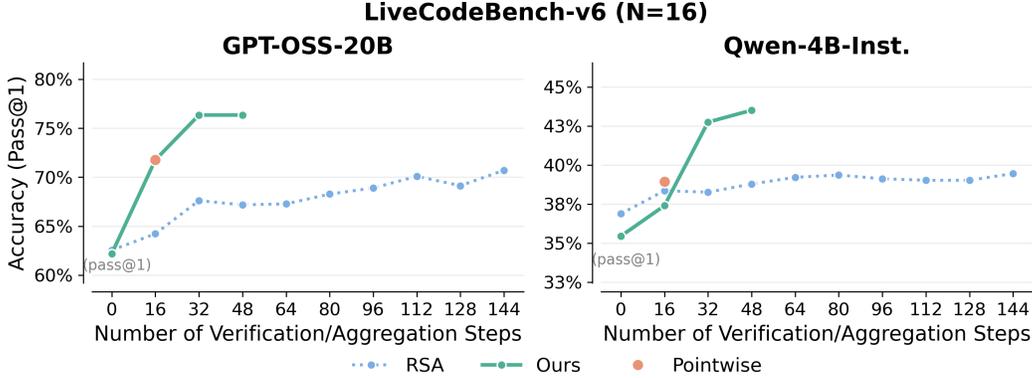
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LiveCodeBench-v6 (N=16)

Figure 5: Comparison with Recursive Self-Aggregation (RSA) [43] on LCB-v6. $\mathbf{V}_1$-Infer achieves higher accuracy with fewer model calls.

Uncertainty-Guided Score Aggregation. Standard pairwise voting often treats all "wins" equally, and thus fails to distinguish between a marginal preference (e.g., a 6 vs. 5 rating) and a decisive victory. Instead, we prompt the models to score their solutions in a pairwise setting instead of simply providing "correct" or "incorrect" as the output as in standard pairwise voting, which provides us with fine-grained information about the goodness of solutions. To capture this nuance of relative quality, we adopt a weighted aggregation scheme in which the magnitude of the rating difference serves as a proxy for the judge's confidence. Given N candidate solutions $\mathcal{S}$ generated by an LLM, and a comparison budget B (number of self-verification LLM calls), a pairwise comparison between (s_i, s_j) using the same LLM outputs calibrated ratings $(r_i, r_j) \in [1, 10]$ for the two solutions. We define a confidence weight

$$w_{ij} = \max\left(\frac{|r_i - r_j|}{9}, \tau\right),$$

where $\tau > 0$ is a small floor that ensures non-zero weights even for near-ties. Let $v_{ij} \in \{0, 0.5, 1\}$ denote the comparison outcome for s_i, corresponding to a win, tie, or loss, respectively. The estimated quality score μ_i is then computed as the uncertainty-weighted win rate

$$\mu_i = \frac{\sum_{j \in \mathcal{N}(i)} w_{ij} v_{ij}}{\sum_{j \in \mathcal{N}(i)} w_{ij}}, \tag{1}$$

where $\mathcal{N}(i)$ denotes the set of opponents compared against s_i. This formulation ensures that high-confidence judgments dominate the global ranking, while ambiguous comparisons contribute minimally to score variance.

Phase 1: Topology Coverage. A primary failure mode in low-budget pairwise ranking is *path dependence* in which solutions become "orphaned" or misranked due to insufficient pairwise comparisons with other solutions. We mitigate this by enforcing a minimum degree constraint to ensure all solutions are pairwise self-verified at least a minimum number of times. We begin with random disjoint pairings to guarantee global connectivity ($d_i \geq 1$), ensuring every solution enters the tournament. Subsequently, we iteratively target under-sampled nodes ($d_i < d_{\min}$) to meet a minimum degree threshold. Rather than selecting random opponents, we pair these low-degree nodes with candidates having the closest current mean score μ. This "anchors" solutions against comparable peers early in the process, preventing initial noise from propagating into the refinement phase.

Phase 2: Swiss Refinement. With the topology anchored, the remaining budget focuses on resolving rank ambiguity through an uncertainty-aware Swiss system. In each round, solutions are sorted by their current score μ, and we pair neighbors within a local window to minimize the score gap $|\mu_i - \mu_j|$ among unseen pairs.

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Algorithm 1 Uncertainty-Guided Pairwise Ranking

Require: Problem x, candidates $\mathcal{S} = \{s_i\}_{i=1}^N$, pairwise budget B, min-degree $d_{\min}$, Swiss window size h, weight floor τ
Ensure: Ranking π

1: **Initialize:** $\forall i$, scores $\mu_i \leftarrow 0.5$ and degrees $d_i \leftarrow 0$
2:     history $\mathcal{H} \leftarrow \emptyset$ $\triangleright (i,j) \in \mathcal{H}$ iff (s_i, s_j) has been compared
3:     state $\mathcal{T} \leftarrow (\{\mu_i\}_{i=1}^N, \{d_i\}_{i=1}^N, \mathcal{H})$
4: used $\leftarrow 0$
    Phase 1: Topology Coverage
5: **while** used $< B$ **and** $\exists i : d_i < d_{\min}$ **do**
6:     $\mathcal{P} \leftarrow \text{CoveragePairs}(\mathcal{T}, d_{\min})$
7:     $\mathcal{O} \leftarrow \text{PairSelfVerify}(x, \mathcal{S}, \mathcal{P})$ $\triangleright$ parallel LLM judging
8:     $\text{UpdateStats}(\mathcal{T}, \mathcal{O}, \tau)$ $\triangleright$ update μ_i, d_i; record $\mathcal{H}$
9:     used $\leftarrow$ used $+|\mathcal{P}|$
10: **end while**
    Phase 2: Swiss Refinement
11: **while** used $< B$ **and** $N > 2$ **do**
12:     $\pi \leftarrow \text{Rank}(\{\mu_i\}_{i=1}^N)$ $\triangleright$ descending μ
13:     $\mathcal{P} \leftarrow \text{SwissPairs}(\pi, \mathcal{T}, h)$ $\triangleright$ within window h; prefer unseen and near-ties
14:     $\mathcal{O} \leftarrow \text{PairSelfVerify}(x, \mathcal{S}, \mathcal{P})$
15:     $\text{UpdateStats}(\mathcal{T}, \mathcal{O}, \tau)$
16:     used $\leftarrow$ used $+|\mathcal{P}|$
17: **end while**
18: **return** $\text{Rank}(\{\mu_i\}_{i=1}^N)$

This strategy is grounded in active learning principles: under Bradley-Terry models, comparisons between items of similar skill (near-ties) yield the highest marginal information gain. By concentrating the judge budget on these ambiguous decision boundaries, $\mathbf{V}_1$-Infer efficiently reduces uncertainty where it matters most, achieving high ranking accuracy even with a sparse comparison graph ($K \ll N$). See Appendix §D for the detailed algorithm and full specification of the helper procedures.

4.1 Experimental Settings

Models and Benchmarks. We evaluate four diverse models: GPT-OSS-20B, Qwen3-4B-Instruct, GPT-OSS-120B, and Qwen-4B-Thinking. For code generation, we use LiveCodeBench-v5 (279 problems between date range of 24.08 to 25.02), LiveCodeBench-v6 (131 problems between 25.02-25.05 following official Qwen3-2507 models[2]) [9], and CodeContests (165 problems) [15]. For real-world software engineering, we evaluate on SWE-bench Lite [11] (300 instances) using Gemini-2.5-Flash [5] as both the generation and verification model. For math, we use AIME'25 and HMMT'25 [1].

Evaluation Protocol and Baselines. For all methods, we first generate N candidate solutions independently from the model, then apply the verification strategy to select the final answer. We use $N \in \{8, 16\}$ for most experiments, and additionally $N = 32$ for pointwise verification in budget-matched comparisons. For comparison with RSA [43] we keep the population size as $N = 16$ with $k = 4$ as the aggregation size (number of solutions to aggregate at a time), with $T = 10$ RSA steps (number of recursive steps that the RSA algorithm is run for). See Venkatraman et al. [43] for a detailed explanation of these parameters. Details about the parameters and settings for inference sampling and swiss-refinement algorithm are provided in Appendix C.

Pointwise Baseline. For fair comparison, pointwise self-verification uses the same 1–10 grading system as our pairwise method. The model evaluates each solution in isolation by assigning a score between 1 and 10, and the solution with the highest score is selected. This ensures that any performance differences stem from the pairwise comparison structure rather than the scoring mechanism. The exact prompts for both pointwise and pairwise verification (for code and math) are provided in Appendix F.

Verification Budget. $\mathbf{V}_1$-Infer allows flexible control over the verification budget. We report results for budget multipliers of $1\times$, $2\times$, and $3\times$ the number of initially generated solutions (N). For example, with $N = 16$ and budget $2\times$, we perform 32 pairwise comparisons.

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[2]https://huggingface.co/Qwen/Qwen3-4B-Instruct-2507

4.2 Results

Our goal is to measure the efficacy of V_1-Infer compared to standard LLM-as-a-judge (pointwise) verification for parallel reasoners, and analyze how it compares with pointwise and aggregation-based test-time scaling methods in a budget-matched setting. Figures 3, 4, and 5 provide an overview of our results, comparing pointwise verification with pairwise verification (Figure 3), budget-matched evaluation (Figure 4), and budgeted comparison with Recursive Self-Aggregation (Figure 5).

V_1-Infer consistently outperforms pointwise self-verification. On CodeContests, GPT-OSS-20B improves from 66.06% to 73.33% (+7.3%), while Qwen3-4B-Instruct improves from 39.4% to 46.1% (+6.7%). On LiveCodeBench-v5, GPT-OSS-20B gains +8.6% and Qwen3-4B-Instruct gains +4.3%. On HMMT, GPT-OSS-20B gains +10.0% and Qwen3-4B-Instruct gains +6.7%. See Figure 3 for more results and Appendix Figure 11 for results on Qwen-4b-Thinking-2507 and GPT-OSS-120B. These results demonstrate that pairwise comparisons provide more informative signals than independent pointwise scores. We further observe that as verification budget increases, pairwise verification performance improves or stays consistent across most models and benchmarks (see Appendix Figure 11). While the results above are for the same number of base solution generations (N=16), we find similar trends while comparing methods in a compute-matched setting as well, Figure 4, e.g., with a compute budget of 64 model calls, with Qwen-4B-Instruct-2507, pointwise verification performs approx. 33% while pairwise verification reaches 45% accuracy on average on code gen. benchmarks.

V_1-Infer enables improved test-time scaling. We compare against Recursive Self-Aggregation (RSA) [43], a state-of-the-art test-time scaling method that iteratively refines solutions through an evolutionary self-aggregation process. As shown in Figure 5, V_1-Infer achieves higher accuracy with significantly fewer LLM calls. On LiveCodeBench-v6 with N=16, our method reaches 76% Pass@1 with only 48 verification calls, higher compared to the maximum accuracy attained by RSA. This efficiency stems from our uncertainty-guided Swiss refinement, which concentrates comparisons on informative pairs rather than exhaustively aggregating or verifying all solution pairs. See Figures. 12, 13, 14 for extended results.

Generalization to real-world software engineering tasks. To evaluate whether pairwise self-verification extends beyond competitive programming and math to open-ended software engineering, we apply V_1-Infer to SWE-bench Lite [11], a benchmark of 300 real GitHub issues from popular Python repositories. For each instance, we use Gemini-2.5-Flash to generate $N=8$ candidate patches via an agentic coding pipeline (mini-swe-agent [50]), then apply self-verification to select the final patch. Crucially, the verification step receives *only* the issue description and the candidate patch diffs. No repository context, execution feedback, or agent trajectory is provided, making this a pure self-verification setting. As shown in Figure 6, pairwise verification achieves a 33.3% resolve rate compared to 28.3% for pointwise and 26.3% for vanilla (first-candidate) selection, an absolute gain of +5.0% over pointwise and +7.0% over vanilla. This result demonstrates that head-to-head patch comparison enables the verifier to identify subtle correctness differences, such as proper root-cause fixes versus surface-level changes, that are difficult to assess in isolation. See Appendix G for detailed examples showing how pairwise and pointwise verification select different patches across diverse bug categories.
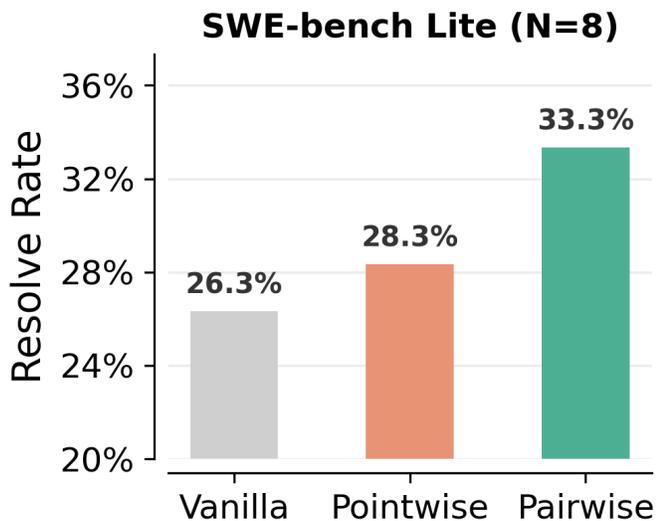

Figure 6: Pairwise comparison with vanilla solution generation and pointwise self-verification on SWE-bench Lite (300 instances, $N=8$ candidates, Gemini 2.5 Flash, mini-swe-agent [50] pipeline).

4.3 Analysis and Ablations

Performance gains across different difficulty levels. Figure 7 shows how $\mathbf{V}_1$-Infer improves accuracy across different problem difficulty levels. On easy problems, Pass@1 is already near-optimal (99.3%). However, on hard problems where Pass@1 is only 40.2%, pairwise verification with budget 3x achieves 63.9%, a gain of +23.7%. Medium difficulty problems show an improvement of +15.4%. This pattern demonstrates that $\mathbf{V}_1$-Infer is most valuable precisely where it is needed most: on challenging problems where the model generates a diverse set of candidate solutions and accurate selection among them is critical for bridging the gap between Pass@1 and Pass@N.

$\mathbf{V}_1$-Infer outperforms random pairwise verification. To validate the effectiveness of our uncertainty-guided refinement algorithm, we compare against a baseline that randomly selects pairs for pairwise comparison. On LCB-v6 with GPT-OSS-20B at budget $3\times$, $\mathbf{V}_1$-Infer achieves 76.3% accuracy compared to 72.5% for random pairing, a gain of +3.8%. This demonstrates that the strategic pair selection in $\mathbf{V}_1$-Infer, which prioritizes comparisons be-
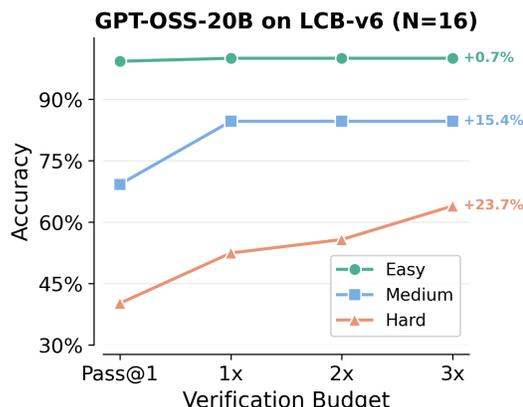

Figure 7: Accuracy improvement with increasing verification budget across problem difficulty levels (GPT-OSS-20B on LCB-v6, N=16). $\mathbf{V}_1$-Infer provides the largest gains on hard problems (+23.7%).

tween solutions with similar quality scores, yields more informative judgments than random sampling.

> **Takeaway: Improved Self-Verification and Test-Time Scaling with $\mathbf{V}_1$-Infer**
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> Pairwise self-verification ($\mathbf{V}_1$-Infer) yields more accurate candidate ranking than pointwise scoring and enables improved performance by test-time scaling of verification compute, compared with self-aggregation based test-time scaling.

5 $\mathbf{V}_1$-PairRL: Improving Self-Verification via Unified RL Training

The previous section demonstrated that pairwise self-verification significantly outperforms pointwise approaches at inference time. In the remainder of the paper we consider a second question: *can we explicitly train models to become stronger self-verifiers?* Current RL paradigms for reasoning focus almost exclusively on optimizing the generation of correct solutions, treating verification as either an afterthought or an external process. While recent work has explored co-training generators with verifiers [18, 32], these approaches rely on *pointwise* rewards and fail to leverage the parallel responses that techniques like GRPO naturally generate during training. Additionally, pointwise verification is uncalibrated, which may make optimization hard. Other methods train for aggregation awareness [43, 55] but use offline data, limiting the model's ability to adapt to its own evolving generation distribution during RLVR training.

We propose $\mathbf{V}_1$-PairRL, a unified RL framework that trains a single LLM to be both a strong reasoner and an accurate *pairwise* self-verifier. The key insight is that generation and verification should *co-evolve*: as the generator improves, the distribution of responses changes, and the verifier must learn score increasingly high-quality solutions. This online, co-evolving setup ensures verification training data is always in-distribution for the model's current capabilities. However, unified training introduces unique challenges: naive implementations suffer from reward hacking, where the generator and verifier collude to maximize reward without improving actual capabilities. We address these challenges through careful reward design and pairing strategies.
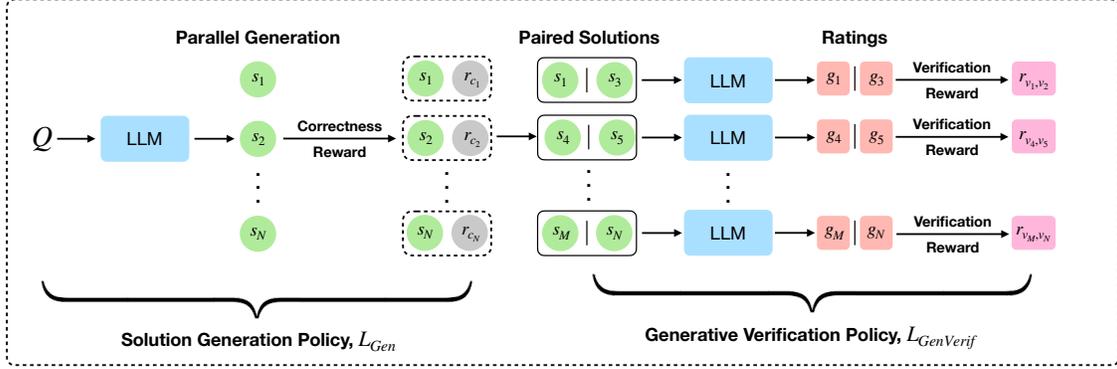
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Figure 8: **V$_1$-PairRL: Unified RL training for co-evolving generation and pairwise verification.** A single LLM is trained with two objectives: J_{Gen} optimizes solution generation using correctness rewards, while $J_{\text{PairVerif}}$ optimizes pairwise verification accuracy. The generator produces G solutions per problem, which are evaluated for correctness and paired for verification training. The verifier evaluates solutions in a paired setting by providing correctness scores for both solutions. Since we are in a verifiable setting and know from ground truth (such as test case execution during code generation) which of the generator solutions are correct, we can calculate correctness rewards for the verifier as well. See Section 5 for more details on how optimization proceeds and rewards are calculated in this framework.

5.1 Preliminaries: RL for LLMs

The goal of RL for LLMs is to maximize expected reward r of an LLM policy π_θ over a distribution of prompts $P(Q)$. This is commonly done with policy gradient methods. Specifically, Group-Relative Policy Optimization (GRPO) [6] and its variants [51] have shown significant promise and stable optimization dynamics. In each episode, we sample prompts $q \sim P(Q)$ and a group of G rollouts per prompt $\{o_i\} \sim \pi_{\text{old}}(.|q)$. We maximize $J_{\text{Gen}}(\theta)$:

$$\mathbb{E}_{q,\{o_i\}} \left[\frac{1}{\sum_{i=1}^{G} |o_i|} \sum_{i=1,t=1}^{G,|o_i|} \min\left(\rho_{i,t} A_i, \text{clip}(\rho_{i,t}, 1-\epsilon, 1+\epsilon) A_i \right) \right] \tag{2}$$

where $A_i = r_i - \text{mean}(\mathbf{r})$ is the advantage computed over the group and $\rho_{i,t} = \pi_\theta(o_{i,t}|q, o_{i,<t})/\pi_{\text{old}}(o_{i,t}|q, o_{i,<t})$ is the importance sampling ratio.

5.2 Co-evolving Solver-Verifier Training

Our training objective combines generation and pairwise verification in a unified RL formulation:

$$J(\theta) = J_{\text{Gen}}(\theta) + \lambda J_{\text{PairVerif}}(\theta) \tag{3}$$

J_{Gen} optimizes the likelihood of correct reasoning paths (using GRPO) and $J_{\text{PairVerif}}$ optimizes the accuracy of pairwise ranking judgments. Both objectives use the *same* rollouts: during each training step, the model generates G solutions per problem, which are used both to compute generation rewards and to form solution pairs for verification training. This ensures the verifier always trains on in-distribution data from the current policy. For verification training, we do one LLM call for each input pair of solutions. Figure 8 shows an overview of **V$_1$**-PairRL. While our framework is general, we instantiate experiments on RL for code-generation following DeepCoder [22].

Rewards. For the solution generator, we use a standard binary correctness reward $r_{\text{gen}} \in \{0, 1\}$ based on passing all ground truth test cases. Reward is set to 0 if response formatting is incorrect, i.e., the model does not output the response withing thinking tags as mentioned in the prompt (§E). For the pairwise self-verification objective, the model compares two solutions (s_A, s_B) and outputs a confidence score between 1 and 10, which is normalized to $v_i \in [0, 1]$ for each solution. We reward the verifier based on how well its
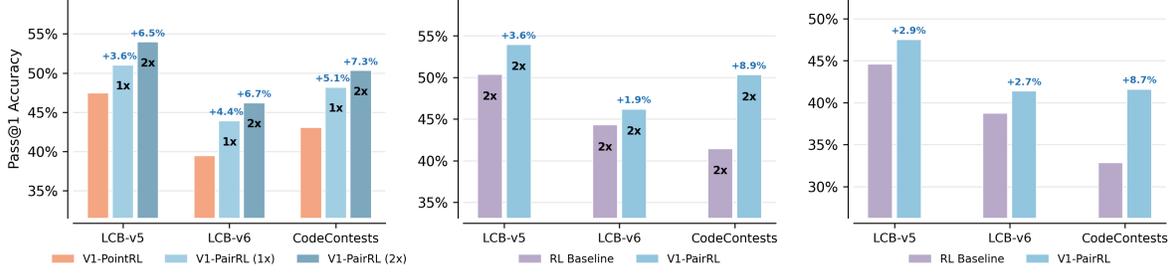
Figure 9: **V₁-PairRL training results (N=16). (left)** Test-time scaling: $\mathbf{V}_1$-PairRL outperforms $\mathbf{V}_1$-PointRL and improves with increased verification budget. **(middle)** With $\mathbf{V}_1$-Infer as the test-time scaling algorithm (at 2x budget): $\mathbf{V}_1$-PairRL outperforms RL baseline even when both use pairwise verification. **(right)** Base Pass@1 accuracy without any test-time scaling: Co-training with pairwise verification improves generation quality over the RL baseline, even without test time scaling.

scores align with ground truth correctness $y_i \in \{0, 1\}$:

$$r_{\text{verif}} = \frac{1}{2} \sum_{i \in \{A, B\}} \mathbb{I}(|v_i - y_i| \le 0.2) \cdot (1 - |v_i - y_i|) \tag{4}$$

The indicator function $\mathbb{I}(|v_i - y_i| \le 0.2)$ implements a *sparsity threshold*: the verifier receives reward only when its score is within 0.2 of the ground truth (i.e., scoring a correct solution ≥ 0.8 or an incorrect solution ≤ 0.2). This design choice is critical for preventing reward hacking, which we discuss below.

Mitigating Reward Hacking. Unified training of generators and verifiers is prone to specific collapse modes. We address two critical forms of reward hacking:

1. *The Safe Bet Collapse:* Without the **sparsity threshold**, the verifier learns to output a safe, middle-ground score (e.g., $v_i = 0.5$) for every solution. This minimizes the risk of being "very wrong" but yields a meaningless discriminator. Sparsity threshold forces the model to commit to confident judgments: only scores near 0 or 1 receive positive reward.

2. *The Empty Solution Loop:* If the verifier is trained on pairs of two incorrect solutions, the generator may collapse into producing empty or trivially incorrect outputs. The verifier easily identifies these as incorrect (scoring them near 0), receiving high reward. This creates a situation where the generator degrades to maximize the verifier's ease of judgment. To prevent this, we enforce a strict pairing strategy: we only trigger verification training when we can form pairs containing at least one correct solution (Correct-Incorrect or Correct-Correct pairs).

5.3 Experimental Settings

We instantiate $\mathbf{V}_1$ on RLVR for code generation following the DeepCoder recipe [22]. Models are trained on the DeepCoder training set, which comprises 24K verified coding problems, utilizing binary rewards determined by executing generated code against ground-truth test cases (1 if all pass, 0 otherwise). We track solution generation performance on the DeepCoder validation set.

Models and Benchmarks. We train Qwen3-4B-Instruct-2507, an instruction-tuned model, following the DeepCoder experimental protocol. We evaluate trained models on LiveCodeBench V5, V6, and CodeContests. See Appendix §C.1 for evaluation hyperparameters.

Pairwise Verification Training. For $J_{\text{PairVerif}}$, we group multiple verification prompts (problem + solution pairs from the solver) rather than generating multiple rollouts for a single prompt. This strategy allows us to leverage information from all pairwise comparisons without increasing the total rollout budget. The advantage for verification rollouts reduces to a REINFORCE-style estimator with a mean baseline calculated across prompts.

Baselines. We compare our approach against two primary baselines: (1) a standard **RL baseline** trained solely for generation without a verification objective, and (2) V_1-**PointRL**, a model jointly trained with pointwise verification rewards under the same setup. Training details for V_1-PointRL are in App. B.

Training Setup. We adopt the DAPO [51] configuration, removing the KL penalty, using Clip High, and applying token-level loss, and follow Dr. GRPO [19] by removing standard deviation normalization. To ensure a fair comparison, we enforce a fixed compute budget of 8 total rollouts per problem. The baseline allocates all 8 rollouts to the solver, while co-evolving models (V_1-PairRL and PointRL) split this budget into 4 solver and 4 verifier rollouts. We verified that training the baseline for a longer duration with fewer rollouts does not improve performance, confirming that gains stem from the co-training objective. All models are trained for 150 steps, with checkpoints selected based on the validation accuracy (pass@1). Prompts used for code-generation training are in Section E.

5.4 Results

V_1-PairRL Offers Superior Test-Time Scaling Capabilities. To evaluate the test-time scaling benefits of co-trained verification, we compare V_1-PairRL against V_1-PointRL, our pointwise verification baseline trained with the same co-evolving setup (as described in Appendix B). As shown in Figure 9(a), pairwise self-verification consistently outperforms pointwise across all benchmarks at N=16. On LiveCodeBench-v5, V_1-PairRL achieves 53.9% (2x budget) compared to 47.4% for V_1-PointRL (+6.5%). Similar improvements are observed on LiveCodeBench-v6 (+6.8%) and CodeContests (+7.3%). V_1-PairRL exhibits positive scaling with verification budget: increasing budget yields consistent accuracy gains across all benchmarks, demonstrating that the model learns to leverage additional pairwise comparisons effectively.

V_1-PairRL outperforms RL Baseline with V_1-Infer as the inference algorithm. In a test-time scaling setup with the same algorithm, a key question arises: does V_1-PairRL show improved performance compared to the standard RL baseline? To investigate, we apply V_1-Infer at inference time to both V_1-PairRL and the RL baseline, using identical 2x verification budgets. As shown in Figure 9 (middle), V_1-PairRL consistently outperforms the RL baseline even when both leverage pairwise verification: +3.6% on LiveCodeBench-v5, +1.9% on LiveCodeBench-v6, and +8.9% on CodeContests. This shows that co-training with pairwise verification enhances overall performance in an equivalent test-time scaling setup.

V_1-PairRL improves RL Baseline. Co-training for pairwise self-verification yields substantial improvements in generation quality. As shown in Figure 9(right), V_1-PairRL achieves consistent Pass@1 improvements over the RL baseline across all three code generation benchmarks at N=16: +2.9% on LiveCodeBench-v5, +2.7% on LiveCodeBench-v6, and +8.7% on CodeContests. The gains demonstrate that jointly optimizing for generation and pairwise verification creates a beneficial learning signal that improves the model's underlying reasoning capabilities (which can also include improving the self-verification capability within long chains of thought), not just its verification accuracy. Our results echo prior findings by Sareen et al. [32], who showed that co-training with pointwise verification improves the base model's Pass@1 performance. We show that pairwise verification takes this one step further: V_1-PairRL also outperforms V_1-PointRL in generation quality, particularly on CodeContests (by about +6%), where the gap is most pronounced (see Appendix Figure 15).

5.5 Ablations

Co-evolving training is critical for inducing strong pairwise self-verification capabilities. V_1-PairRL trains solution generation and verification capabilities in a co-evolving setup where the model is trained for self-verification using samples generated online at the same iteration thus adapting to its own evolving generation distribution. An alternative way of jointly optimizing these capabilities is to train the model in a multi-task setup where the data
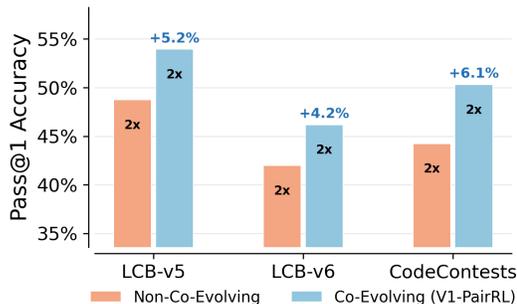

Figure 10: Co-evolving vs. non-co-evolving pairwise verification. Both use V_1-Infer (2x budget).

for the verification task is generated offline, using the base model, before training. To isolate the effect of co-evolving training, we train a *non-co-evolving* baseline: the same model trained with RL for generation as well as verification. For this, we first run the Qwen-4B-Instruct-2507 model on the DeepCoder dataset prompts to generate 8 solutions per prompt, and then use these solutions as part of the verifier prompt during RL training. We apply $\mathbf{V}_1$-Infer at inference time with the same 2x budget. As shown in Figure 10, co-evolving training consistently outperforms the non-co-evolving setup across all benchmarks.

> **Takeaway: Co-Training Generation and Pairwise Self-Verification**
>
> Unified RL training that jointly optimizes generation and pairwise verification ($\mathbf{V}_1$-PairRL) produces stronger reasoning models and improves test-time scaling compared with generation-only baselines or models co-trained with pointwise verification.

6   Conclusion

We presented $\mathbf{V}_1$, a unified framework for advancing self-verification in parallel reasoning, grounded in the insight that pairwise comparison constitutes a fundamentally more robust primitive for verification than absolute scoring. We introduced $\mathbf{V}_1$-Infer, which employs tournament-based refinement to dynamically allocate compute toward ambiguous pairs, significantly outperforming pointwise verification and prior aggregation techniques with lower computational overhead. We further improved these results by complementing $\mathbf{V}_1$ with a post-training approach, $\mathbf{V}_1$-PairRL, which demonstrated that jointly training a single model for both generation and pairwise self-verification unlocks superior test-time scaling compared to standard RL and pointwise baselines. By unifying generation with pairwise verification, $\mathbf{V}_1$ provides a robust framework for both effective RL training and scalable parallel reasoning.

Acknowledgements

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A Extended Related Work

Parallel reasoning and test-time scaling. Test-time scaling approaches fall into two paradigms: sequential methods such as long chain-of-thought that iteratively refine a single solution [23], and parallel methods that generate multiple reasoning paths simultaneously [4, 17, 29, 33, 37, 54]. Parallel scaling helps explore diverse

solution paths simultaneously; however, it requires effective mechanisms to combine various solutions. For math, majority voting is effective due to objective answers [38, 44, 48], but this approach is domain-specific and unavailable for domains such as code generation and scientific discovery, where answers are not directly comparable to each other via exact matching. For code generation, prior work [14] has explored test-time scaling but requires executable test cases in the evaluation data or generated by larger models like GPT-4 for clustering and selection, while μCode [8] leverages external, multi-turn execution feedback, both of which are specific to code generation. We focus on *self-verification* for parallel reasoners, where a model judges its own generations without external verified or feedback.

Self-verification and self-aggregation Early work by Weng et al. [46] demonstrated that LLMs can verify their own chain-of-thought reasoning, improving accuracy on arithmetic and commonsense tasks. Related to this, there have been multiple studies analysing the self-refinement capabilities and limitations of LLMs [7, 23, 39], however, they focus on a sequential reasoning setting, while our focus is on explicit self-verification in parallel reasoning. Recently Lu et al. [21] show that LLMs exhibit a bias toward accepting incorrect solutions during pointwise self-verification. On the other hand, self-aggregation methods like RSA [43] and parallel-distill-refine [24] combine solutions generated by the same model, but suffer from diversity collapse (Section 4), resulting in correct solutions being discarded during inference. Such aggregation-based approaches are orthogonal to self-verification and can be combined with better self-verification for more better performance and efficient test-time scaling. Our work addresses limitations of the above prior work by studying pairwise self-verification in a parallel reasoning setup, showing it significantly outperforms pointwise self-verification while preserving solution diversity.

Generative verifiers and reward models. Reward models have been central to LLM alignment and RLHF [3, 40, 58], and have emerged as a natural mechanism for scoring and selecting LLM generations [4, 56]. Recent work demonstrates that generative reward models, which produce chain-of-thought reasoning before scoring, substantially outperform discriminative approaches [25, 31, 36, 53], with RL-trained verifiers showing improved judgment performance [20, 47]. Recognizing that pairwise comparison is often easier than absolute scoring, several works have explored pairwise reward models: PairRM [10] trains a discriminative pairwise ranker for ensembling LLM outputs, while GenSelect [42] and concurrent work by Mahdavi et al. [26] scale generative verification for mathematical reasoning. Related to this, Zhao et al. [55] propose AggLM, which trains an aggregator via RL to synthesize correct answers for improving majority voting. However, these approaches use *separate* verifiers or aggregator models. Our work differs in two key aspects: (1) we study *self*-verification, where the same parallel reasoner model judges its own outputs. This eliminates the need for additional training with curated judge data, saves memory, and reduces the computational overhead associated with external verification. (2) We co-train the model to be both a solution generator and a pairwise self-verifier in a unified framework.

Co-training for generation and verification. Training separate verifiers incurs significant overhead in compute, memory, and data curation. Recent work has explored co-training a single model for both generation and verification. Sareen et al. [32] train models to self-verify online during RL, ensuring the verifier sees in-distribution generations and uses this capability to perform better majority voting at test-time for math reasoning. Liu et al. [18] extends this to unified CoT verifiers, while Zha et al. [52] co-trains a separate process reward model that provides intermediate rewards. Offline approaches train verifiers or aggregators on data sampled from the base model [30, 43]. For code generation specifically, several works co-train generators with unit-test producers [13, 45]. However, existing co-training methods rely on *pointwise* verification rewards. Our V_1-PairRL framework instead co-trains for *pairwise* self-verification in an online, co-evolving setup, which we show yields superior test-time scaling performance.

18

B V₁-PointRL Training Details

To provide a fair comparison for our pairwise verification approach, we train $\mathbf{V}_1$-PointRL using the same co-evolving setup as $\mathbf{V}_1$-PairRL. The key difference lies in the verification objective: instead of comparing pairs of solutions, $\mathbf{V}_1$-PointRL assigns an independent score to each solution.

Verification Reward. For pointwise verification, the model evaluates a single solution s and outputs a confidence score between 1-10 which is normalized to $v \in [0, 1]$. The reward is computed as:

$$r_{\text{verif}} = \mathbb{I}(|v - y| \le 0.2) \cdot (1 - |v - y|) \tag{5}$$

where $y \in \{0, 1\}$ is the ground truth correctness. The sparsity threshold prevents the "safe bet" collapse mode where the model learns to always output $v = 0.5$ (in practice, not applying the sparsity threshold leads to collapse in solution generation performance and is necessary for stable training).

Training Configuration. All hyperparameters (learning rate, batch size, rollout allocation, etc.) are identical to $\mathbf{V}_1$-PairRL as shown in Table 1. The only difference is the verification prompt and reward computation.

Inference. At inference time, $\mathbf{V}_1$-PointRL generates N solutions and independently scores each one. The solution with the highest pointwise score is selected as the final answer.

C Hyperparameters

C.1 Inference Sampling Parameters

Inference Framework. We use SGLang [57] for all inference experiments, which provides efficient batched inference for large language models.

Sampling Parameters. We set temperature $T = 0.6$ for all code generation experiments and $T = 1.0$ for all math reasoning experiments to encourage better exploration of the solution space. We use top-p sampling with $p = 0.95$ for all experiments. For experiments with $\mathbf{V}_1$-Infer in Section 4 where we test the inference algorithm on widely used open source LLMs like GPT-OSS-20B, GPT-OSS-120B, Qwen-4B-Instruct-2507, Qwen-4B-Thinking-2507, we set the max generation length to 32768. We find that this does not lead to truncation for most evaluation examples, as also found in Venkatraman et al. [43]. We keep the same generation length for $\mathbf{V}_1$-Infer, pointwise verification as well as RSA.

For trained model evaluations in Section 5 we find that training all models lead to longer chains of thoughts, as is commonly the case in RLVR [6]. This leads to frequent truncation, especially for the baseline RL model. To resolve this, we adopt the truncate-and-continue-generation method from prior work [32, 49]. When generation reaches the maximum token budget (`finish_reason=length`), we resume decoding by appending the partially generated assistant output to the message history and issuing a continuation request. If the truncated output does not contain a closing `</thinking>` tag, we append the following transition sentence before continuing: *"Considering the limited time by the user, I have to directly give the required response based on the reasoning till now directly.`</thinking>`"*. This explicitly closes the thinking block and forces the continuation to produce the final answer. If the truncated output already contains `</thinking>`, we directly continue generation without appending any additional text. We continue the generation for 2K tokens, making the total maximum generation length 34 K for all experiments in Section 5.

Number of Samples. We generate $N \in \{8, 16\}$ candidate solutions for most experiments. For budget-matched comparisons between pointwise and pairwise verification, we additionally evaluate pointwise verification with $N = 32$ samples to match the total LLM calls of pairwise verification with $N = 16$ and budget $3\times$. This budget-matched comparison is performed only for GPT-OSS-20B and Qwen3-4B-Instruct models.

Verification Budget. For $\mathbf{V}_1$-Infer experiments in Section 4, we evaluate verification budgets of $1\times, 2\times$, and $3\times$ the number of base solutions (N). Full results across all budgets are provided in the appendix figures.

Experimental Runs. All inference experiments for base models (Section 4) are run once. All trained model results (Section 5) are run 3 times with different random seeds, and we report the mean. Due to the computational cost of running multiple seeds, we limit the budget exploration for trained models to $1\times$ and $2\times$, though we believe performance can further scale with additional budget for the pairwise method ($\mathbf{V}_1$-Infer).

Swiss Refinement Algorithm Parameters. For $\mathbf{V}_1$-Infer, we use the following settings: Minimum degree $d_{\min} = 2$ (each solution compared at least twice in coverage phase), Swiss window size $h = 8$, Confidence floor set to a small value, $\tau = 0.1$ for weighted aggregation. We do not tune parameters to any specific benchmark. We use the same parameters across benchmarks or models across all experiments.

C.2 Training Hyperparameters

Table 1 summarizes the key training hyperparameters. We train models using rLLM [41] and verl [35] backend.

Table 1: Training hyperparameters for $\mathbf{V}_1$.

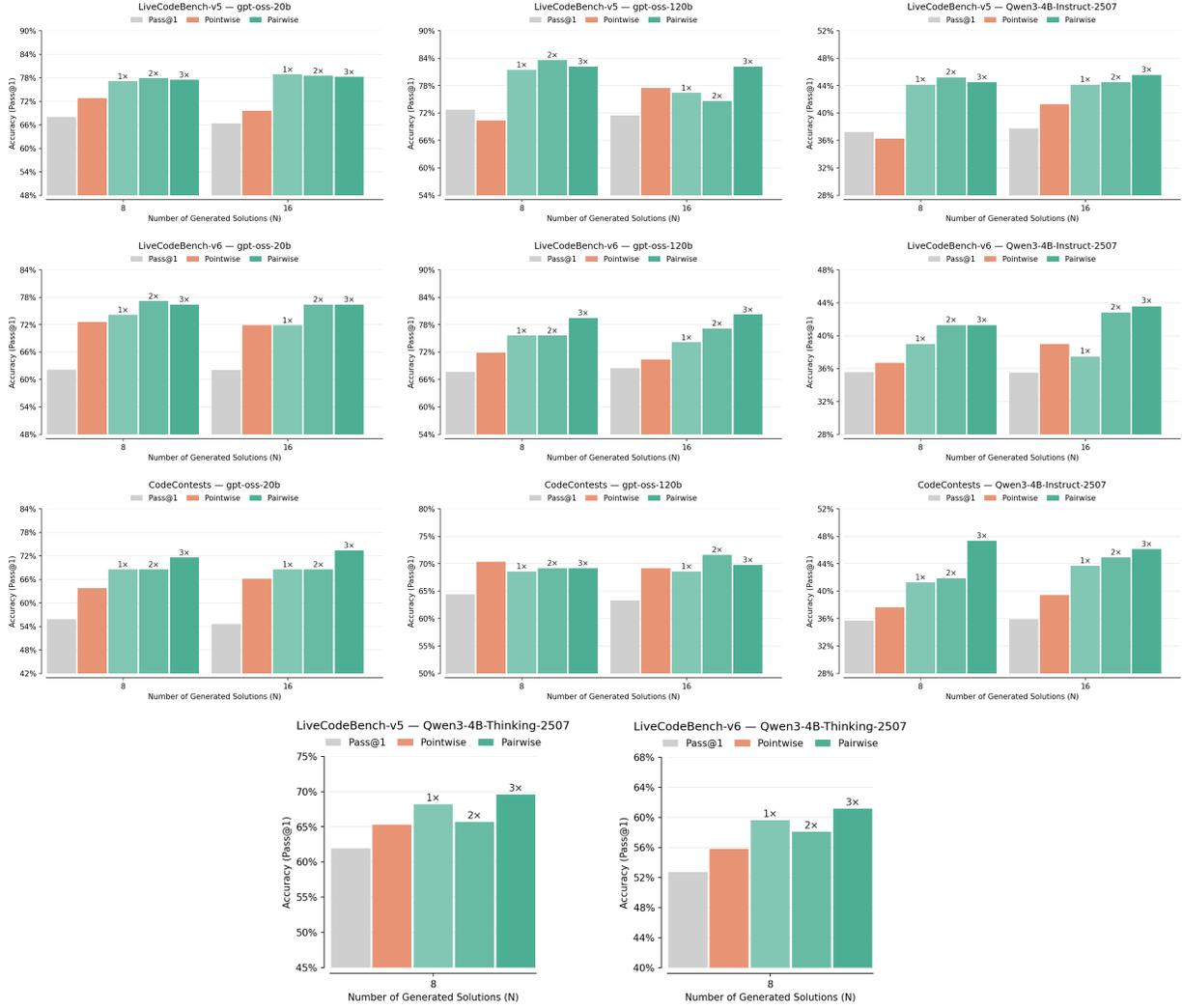| Hyperparameter	Qwen3-4B-Inst
Learning rate	1×10^{-6}
Batch size	64
Rollouts per prompt (G)	4 (8 for baseline RL)
Judge rollouts	4 (0 for baseline RL)
Max prompt length	10240
Max response length	24576
Temperature	0.6
Top-p	0.95
Clip ratio (low)	0.2
Clip ratio (high)	0.28
λ (loss weight)	1.0
KL coefficient	0
Entropy coefficient	0
Std normalization	No
Loss aggregation	token-mean

Figure 11: Self-verification bar plots across benchmarks and models (LiveCodeBench-v5, LiveCodeBench-v6, and CodeContests). Each plot compares Pass@1, pointwise verification, and pairwise verification (budgets $1\times/2\times/3\times$) at $N \in \{8, 16\}$. For Qwen3-4B-Thinking (bottom row), we run experiments only on Live-CodeBench at $N = 8$ to manage compute.

Figure 12: **Comparing Pairwise Verification with RSA.** Top row: LiveCodeBench-v6. Bottom row: LiveCodeBench-v5. Columns (left→right): gpt-oss-20b, Qwen3-4B-Instruct-2507. **Here number of generations by the base model is 16**, followed by verification (pointwise, pairwise) or aggregation (RSA).
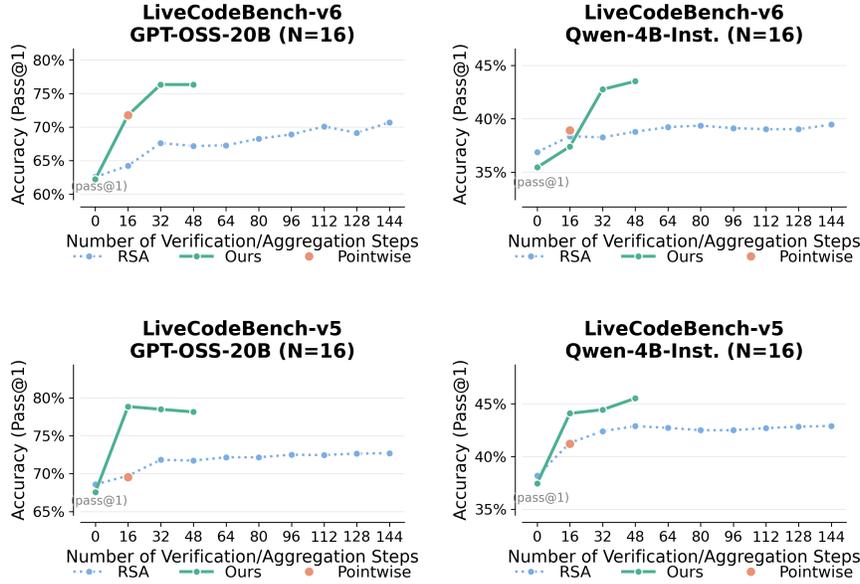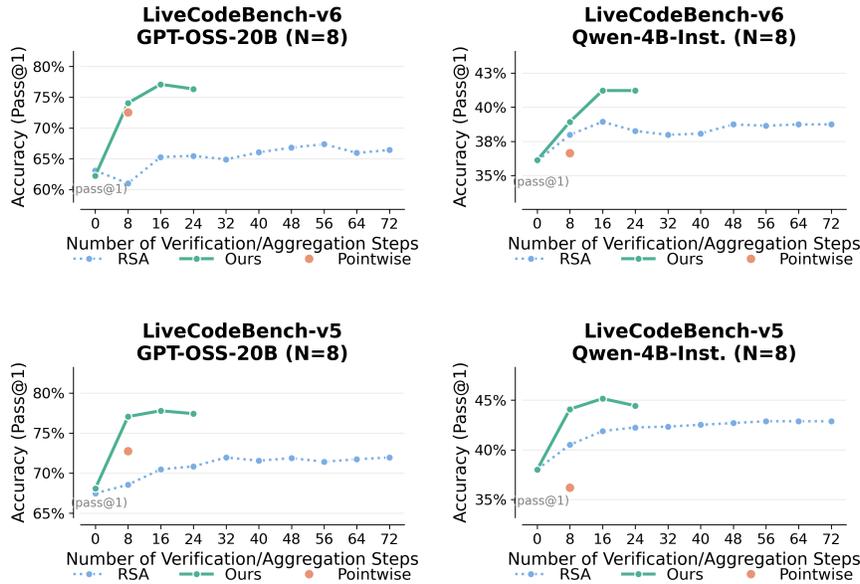

Figure 13: **Comparing Pairwise Verification with RSA.** Top row: LiveCodeBench-v6. Bottom row: LiveCodeBench-v5. Columns (left→right): gpt-oss-20b, Qwen3-4B-Instruct-2507. **Here number of generations by the base model is 8**, followed by verification (pointwise, pairwise) or aggregation (RSA).
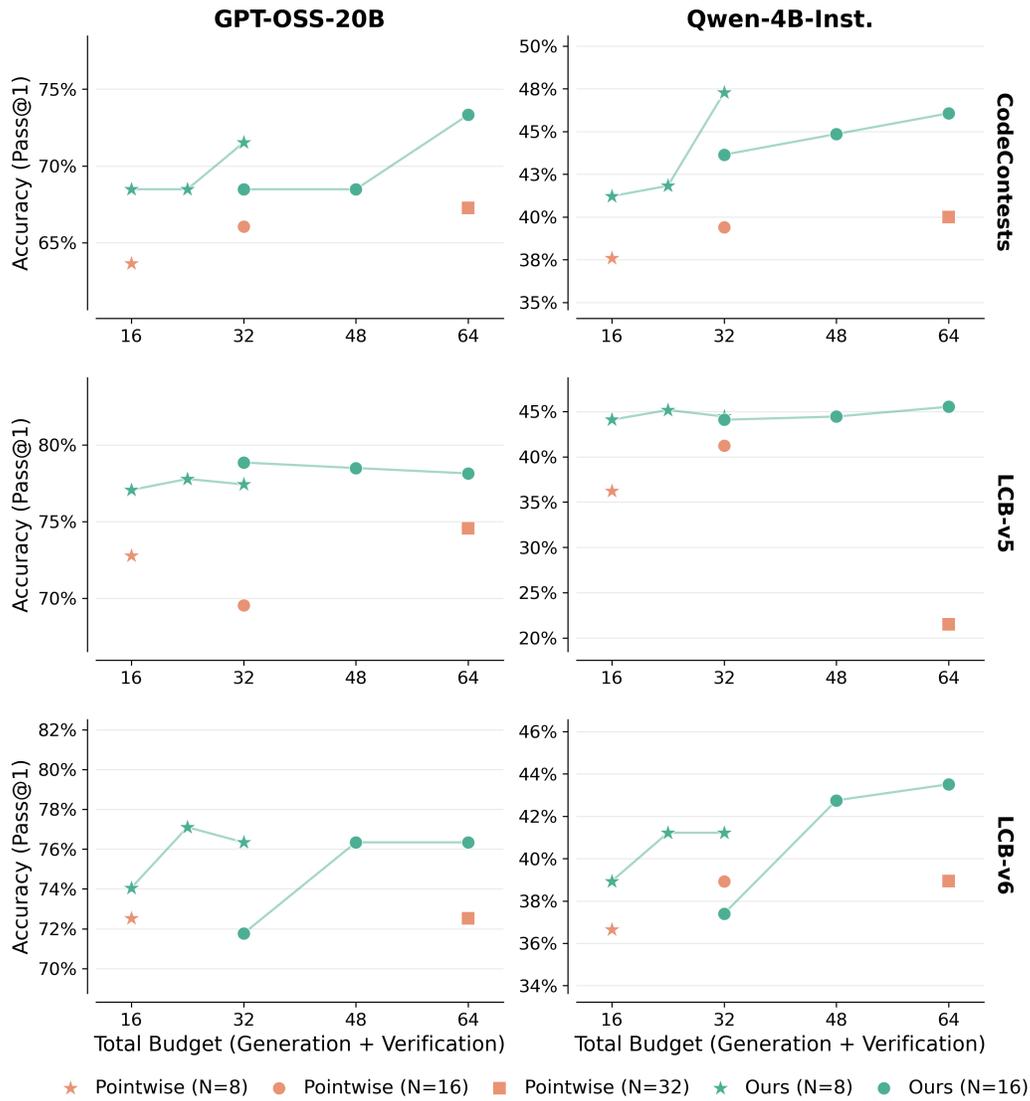
Figure 14: **Budget vs accuracy per benchmark.** Accuracy vs. total budget (generation + verification calls) for CodeContests (top row), LiveCodeBench-v5 (middle row), and LiveCodeBench-v6 (bottom row). Left column: GPT-OSS-20B. Right column: Qwen-4B-Inst. Stars denote $N = 8$ base generations, circles denote $N = 16$, squares denote $N = 32$ (pointwise only).

D   Detailed Algorithm

This section provides the full specification of the helper procedures used in Algorithm 1. The main paper presents a high-level view for clarity; here we give the exact update rules and pairing strategies required for reproducibility.

State.   We maintain a state $\mathcal{T} = (\{\mu_i\}_{i=1}^N, \{d_i\}_{i=1}^N, \mathcal{H})$, where μ_i is the current estimated quality score of solution s_i, d_i is the number of distinct opponents it has been compared against, and $\mathcal{H} \subset [N] \times [N]$ records previously compared pairs.

CoveragePairs.   Ensures minimum degree while anchoring comparisons to similar-quality peers.

Algorithm 2 COVERAGEPAIRS($\mathcal{T}, d_{\min}$)

Require: State $\mathcal{T} = (\{\mu_i\}, \{d_i\}, \mathcal{H})$, minimum degree $d_{\min}$
Ensure: Disjoint pair set $\mathcal{P}$
 1: $\mathcal{P} \leftarrow \emptyset$, $\mathcal{U} \leftarrow \emptyset$ ▷ used indices
 2: $\mathcal{L} \leftarrow \{i : d_i < d_{\min}\}$; sort $\mathcal{L}$ by increasing d_i
 3: **for** each $i \in \mathcal{L}$ **do**
 4:     **if** $i \in \mathcal{U}$ **then**
 5:         **continue**
 6:     **end if**
 7:     $\mathcal{C} \leftarrow \{j \neq i : j \notin \mathcal{U},\ (i,j) \notin \mathcal{H}\}$
 8:     **if** $\mathcal{C} = \emptyset$ **then**
 9:         **continue**
10:     **end if**
11:     $j \leftarrow \arg\min_{j \in \mathcal{C}} |\mu_i - \mu_j|$
12:     $\mathcal{P} \leftarrow \mathcal{P} \cup \{(i,j)\}$
13:     $\mathcal{U} \leftarrow \mathcal{U} \cup \{i,j\}$
14: **end for**
15: **return** $\mathcal{P}$

UpdateStats.   This is where the mathematical aggregation is implemented. Note. In practice, the update

---

Algorithm 3 UPDATESTATS($\mathcal{T}, \mathcal{O}, \tau$)

Require: State $\mathcal{T} = (\{\mu_i\}, \{d_i\}, \mathcal{H})$, outcomes $\mathcal{O}$, floor $\tau > 0$
Ensure: Updated state $\mathcal{T}$
 1: **for** each $(i, j, w, r_i, r_j) \in \mathcal{O}$ **do**
 2:     $v_{ij} \leftarrow \mathbb{I}[w = \mathtt{A}] + 0.5 \cdot \mathbb{I}[w = \mathtt{tie}]$
 3:     $w_{ij} \leftarrow \max\left(\frac{|r_i - r_j|}{9}, \tau\right)$
 4:     update μ_i, μ_j using Eq. (1)
 5:     **if** $(i,j) \notin \mathcal{H}$ **then**
 6:         $d_i \leftarrow d_i + 1; \quad d_j \leftarrow d_j + 1$
 7:         $\mathcal{H} \leftarrow \mathcal{H} \cup \{(i,j), (j,i)\}$
 8:     **end if**
 9: **end for**

---

of μ_i is implemented incrementally using sufficient statistics, but Eq. (1) fully specifies the aggregation semantics.

SwissPairs. Allocates remaining budget to the most informative near-ties.

Algorithm 4 SwissPairs($\pi, \mathcal{T}, h$)

Require: Ranking π, state $\mathcal{T} = (\{\mu_i\}, \{d_i\}, \mathcal{H})$, window size h
Ensure: Disjoint pair set $\mathcal{P}$
1: $\mathcal{P} \leftarrow \emptyset$, $\mathcal{U} \leftarrow \emptyset$
2: **for** $k = 1$ **to** N **do**
3:     $i \leftarrow \pi_k$
4:     **if** $i \in \mathcal{U}$ **then**
5:         **continue**
6:     **end if**
7:     $\mathcal{C} \leftarrow \emptyset$
8:     **for** $t = k + 1$ **to** $\min(k + h, N)$ **do**
9:         $j \leftarrow \pi_t$
10:         **if** $j \in \mathcal{U}$ **then**
11:             **continue**
12:         **end if**
13:         add $(|\mu_i - \mu_j|, \mathbb{I}[(i,j) \in \mathcal{H}], j)$ to $\mathcal{C}$
14:     **end for**
15:     **if** $\mathcal{C} \neq \emptyset$ **then**
16:         choose j with lexicographically minimal key in $\mathcal{C}$
17:         $\mathcal{P} \leftarrow \mathcal{P} \cup \{(i,j)\}$
18:         $\mathcal{U} \leftarrow \mathcal{U} \cup \{i, j\}$
19:     **end if**
20: **end for**
21: **return** $\mathcal{P}$

The above procedures jointly ensure (i) early stabilization of the comparison topology via minimum-degree coverage and (ii) efficient use of remaining budget through uncertainty-focused Swiss refinement, enabling accurate ranking with $O(N)$–scale pairwise verification.

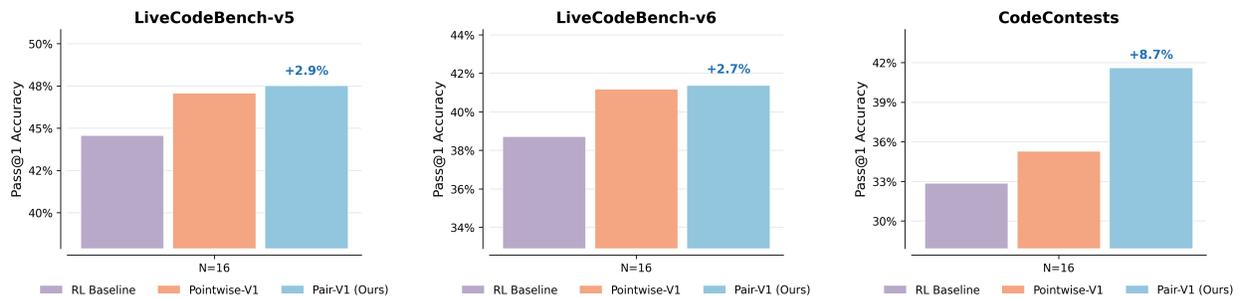

Figure 15: **V$_1$-PairRL vs RL baseline across benchmarks (N=16).** Left: LiveCodeBench-v5. Middle: LiveCodeBench-v6. Right: CodeContests. Co-training with pairwise verification consistently improves Pass@1 accuracy across all benchmarks.

E   Generation Prompts

E.1   Code Generation

> **Code Generation Prompt**
>
> ```
> **Problem**
> {problem}
>
>
> Think and reason step by step before coding the final solution for the problem above.  Put
> your reasoning and any draft coding solutions between <thinking> ...  </thinking> tags.
> After the reasoning (i.e.  after the </thinking> tag), use the format provided in the
> problem above (code-block with backticks) to format your final code solution.  Do not
> include any thinking within the code block.
>
>
> ### Answer:
> ```

Note that "Do not include any thinking within the code-block." is added to the prompt, since in the absence of it, training the model (Qwen-4B-Instruct-2507) leads to it generating large amounts of comments within the code block, effectively leading to truncation of the response (due to reaching max length) and hence, 0 rewards and training collapse.

E.2   Math Solution Generation

> **Math Solution Generation Prompt**
>
> ```
> {problem}.  Let's think step by step and output the final answer within \boxed{}.
> ```

F Verification Prompts

This section provides the exact prompts used for pointwise and pairwise self-verification in our experiments. Both verification methods use a 1–10 rating scale for fair comparison.

F.1 Pointwise Verification Prompts

F.1.1 Code Generation

> **Pointwise Code Verification Prompt**
>
> ```
> You are an expert code reviewer.  Rate the correctness of a solution to a programming
> problem.
>
> **Evaluation Guidelines:**
> - Analyze the problem's requirements and constraints.
> - Mentally trace the solution with test cases (including edge cases) to verify correctness.
> - Give a higher score if the solution is robust and fault-tolerant.
>
> **Problem**
> {problem}
>
> **Solution**
> {code}
>
> **Output Format:**
> First, provide your step-by-step reasoning.  Then, on a new line, provide your final rating
> using the EXACT tags below.  Add no other text after the tags.
> <rating>INTEGER_1_TO_10</rating>
>
> **Rating Rules:**
> - Rate correctness on a 1-10 scale (10 = correct & robust, 5 = borderline, 1 = incorrect).
> Please provide your analysis now.
> ```

F.1.2 Math Reasoning

> **Pointwise Math Verification Prompt**
>
> ```
> You are an expert math contest grader.  Rate the correctness of a submission based solely on
> the final answer.
>
> **Evaluation Guidelines:**
> - Extract the submission's final answer.  Use any provided reasoning only to help you assess
> whether the stated final answer is trustworthy.  Do not award credit for method quality or
> rigor.
> - Carefully analyze the problem statement and the submission to assess whether the final
> answer is correct.  Grade only the final answer.
>
> **Problem**
> {problem}
>
> **Solution**
> {solution}
>
> **Output Format:**
> First, provide your reasoning (what checks you performed).  Then, on a new line, provide
> your final rating using the EXACT tag below.  Add no other text after the tag.
> <rating>INTEGER_1_TO_10</rating>
> ```

```
**Rating Rules:**
- Rate correctness on a 1-10 scale (10 = certainly correct, 8 = very likely correct, 5 =
uncertain/borderline, 3 = likely incorrect, 1 = certainly incorrect).
Please provide your analysis now.
```

F.2 Pairwise Verification Prompts

F.2.1 Code Generation

Pairwise Code Verification Prompt

```
You are an expert code reviewer.  Compare two solutions to a programming problem and rate
their correctness.

**Evaluation Guidelines:**
- Analyze the problem's requirements and constraints.
- Mentally trace each solution with test cases (including edge cases) to verify correctness.
- If both solutions appear correct, prefer the more robust and fault-tolerant one.

**Problem**
{problem}

**Solution A**
{code_A}

**Solution B**
{code_B}

**Output Format:**
First, provide your step-by-step reasoning.  Then, on separate new lines, provide your final
ratings using the EXACT tags below.  Add no other text after the tags.
<rating_A>INTEGER_1_TO_10</rating_A>
<rating_B>INTEGER_1_TO_10</rating_B>

**Rating Rules:**
- Rate correctness on a 1-10 scale (10 = correct & robust, 5 = borderline, 1 = incorrect).
- The higher rating wins.  Equal ratings imply a tie.
Please provide your analysis now.
```

F.2.2 Math Reasoning

Pairwise Math Verification Prompt

```
You are an expert math contest grader.  Compare two submissions and rate correctness based
solely on the final answer.

**Evaluation Guidelines:**
- Extract each submission's final answer.  Use any provided reasoning only to help you assess
whether the stated final answer is trustworthy.  Do not award credit for method quality or
rigor.
- Carefully analyze the problem statement and the submissions to assess whether each final
answer is correct.  Grade only the final answer.

**Problem**
{problem}

**Solution A**
{sol_A}
```

```
**Solution B**
{sol_B}

**Output Format:**
First, provide your reasoning (what checks you performed).  Then, on separate new lines,
give ratings using the EXACT tags below.  Add no other text after the tags.
<rating_A>INTEGER_1_TO_10</rating_A>
<rating_B>INTEGER_1_TO_10</rating_B>

**Rating Rules:**
- Rate correctness on a 1-10 scale (10 = certainly correct, 8 = very likely correct, 5 =
uncertain/borderline, 3 = likely incorrect, 1 = certainly incorrect).
- Higher rating wins.  Equal ratings imply a tie.
Please provide your analysis now.
```

G SWE-bench Verification Examples

This section presents representative examples from our SWE-bench Lite evaluation (300 instances, 8 candidate patches each, Gemini 2.5 Flash as both generator and verifier). We show cases where pairwise and pointwise verification select different patches, illustrating how head-to-head comparison can surface correct solutions that independent scoring misses, and vice versa. For each example, we show the issue description, the patches selected by each method, and their rankings.

G.1 Example 1: `django__django-11049`: Pairwise Correct, Pointwise Wrong

> **Issue: Correct expected format in invalid DurationField error message**
>
> If you enter a duration "14:00" into a duration field, it translates to "00:14:00" which is 14 minutes. The current error message for invalid DurationField says that this should be the format of durations: `[DD] [HH:[MM:]]ss[.uuuuuu]`. But according to the actual behaviour, it should be: `[DD] [[HH:]MM:]ss[.uuuuuu]`, because seconds are mandatory, minutes are optional, and hours are optional if minutes are provided. This seems to be a mistake in all Django versions that support the DurationField. Also the duration fields could have a default `help_text` with the requested format, because the syntax is not self-explanatory.

Setup: 8 candidate patches generated, all non-empty.
Pairwise ranking: $[6, 3, 5, 1, 7, 0, 4, 2]$ → selected **idx=6** ✓
Pointwise ranking: $[0, 1, 3, 5, 6, 7, 4, 2]$ → selected **idx=0** ✗

> **Pairwise-Selected Patch (idx=6): Resolves the issue** ✓
>
> ```
> diff -git a/django/db/models/fields/__init__.py
> @@ -1587,7 +1587,7 @@ class DurationField(Field):
> default_error_messages = {
> 'invalid': _("'%(value)s' value has an invalid format.  It must be in "
> -          "[DD] [HH:[MM:]]ss[.uuuuuu] format.")
> +          "[DD] [[HH:]MM:]ss[.uuuuuu] format.")
> }
>
> diff -git a/django/forms/fields.py
> @@ -467,6 +467,8 @@ class DurationField(Field):
> +  help_text = _('Accepted format is [DD] [[HH:]MM:]ss[.uuuuuu]')
> ```

> **Pointwise-Selected Patch (idx=0): Does not resolve the issue** ✗
>
> ```
> diff -git a/django/forms/fields.py
> @@ -466,12 +466,19 @@ class DurationField(Field):
> +  default_help_text = _('Accepted duration format is [[DD] [[HH:]MM:]SS[.uuuuuu]].')
> def __init__(self, *, help_text=None, **kwargs):
> if help_text is None:
> help_text = self.default_help_text
> super().__init__(help_text=help_text, **kwargs)
> ```

Analysis: The correct patch (idx=6) fixes both the error message format string in the model field *and* adds help text to the form field. The pointwise-selected patch (idx=0) only adds help text to the form field but does not fix the incorrect format string in the model's error message. Pairwise comparison enables the verifier to directly contrast these two approaches, surfacing that fixing the error message is the core requirement.

G.2 Example 2: `matplotlib__matplotlib-25311`: Pairwise Correct, Pointwise Wrong

Issue: Unable to pickle figure with draggable legend

Bug summary: Unable to pickle figure with draggable legend. Same error comes for draggable annotations.

Code for reproduction:
```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(time, speed, label="speed")
leg = ax.legend()
leg.set_draggable(True) # pickling works after removing this line
pickle.dumps(fig) # raises TypeError
```

Actual outcome: `TypeError:  cannot pickle 'FigureCanvasQTAgg' object`
Expected outcome: Pickling successful

Setup: 8 candidate patches generated, 7 non-empty.
Pairwise ranking: [6, 2, 3, 4, 5, 1, 0]   → selected **idx=6** ✓
Pointwise ranking: [0, 1, 2, 3, 4, 6, 5]   → selected **idx=0** ✗

Pairwise-Selected Patch (idx=6): Resolves the issue ✓

```
diff -git a/lib/matplotlib/offsetbox.py
@@ -1472,6 +1472,23 @@ class DraggableBase:
+  def __getstate__(self):
+    self.disconnect()
+    state = self.__dict__.copy()
+    state["canvas"] = None
+    state["cids"] = []
+    if "_c1" in state:  del state["_c1"]
+    return state
+  def __setstate__(self, state):
+    self.__dict__.update(state)
```

Pointwise-Selected Patch (idx=0): Does not resolve the issue ✗

```
diff -git a/lib/matplotlib/legend.py
@@ -1204,6 +1204,22 @@ class Legend(Artist):
+  def __getstate__(self):
+    draggable_state = self._draggable is not None
+    if draggable_state:  self.set_draggable(False)
+    state = super().__getstate__()
+    state["_draggable_state"] = draggable_state
+    return state
+  def __setstate__(self, state):
+    draggable_state = state.pop("_draggable_state")
+    super().__setstate__(state)
+    if draggable_state:  self.set_draggable(True)
```

Analysis: The correct patch (idx=6) fixes the problem at its root in `DraggableBase`, the parent class used by both draggable legends and annotations. It properly handles the unpickleable canvas reference. The pointwise-selected patch (idx=0) attempts the fix only in the `Legend` subclass, missing the general case for draggable annotations. The pairwise tournament allowed the verifier to compare these approaches side-by-side, recognizing that the root-class fix is more complete.

G.3 Example 3: `matplotlib__matplotlib-23964`: Pairwise Correct, Pointwise Wrong

> **Issue: PostScript backend crash on empty text with usetex**
>
> When using the PostScript backend with `usetex=True`, rendering an empty string raises an error because `curr_stream` is uninitialized when appended to the stream list after iterating over an empty glyph sequence.

Setup: 8 candidate patches generated, all non-empty.
Pairwise ranking: $[4, 0, 1, 2, 3, 5, 6, 7] \quad \rightarrow$ selected **idx=4** ✓
Pointwise ranking: $[0, 1, 4, 2, 3, 5, 6, 7] \quad \rightarrow$ selected **idx=0** ✗

> **Pairwise-Selected Patch (idx=4): Resolves the issue** ✓
>
> ```
> diff -git a/lib/matplotlib/backends/backend_ps.py
> @@ -666,7 +666,8 @@
>  )
>  # append the last entry
> -        stream.append(curr_stream)
> +        if curr_stream:
> +            stream.append(curr_stream)
> ```

> **Pointwise-Selected Patch (idx=0): Does not resolve the issue** ✗
>
> ```
> diff -git a/lib/matplotlib/backends/backend_ps.py
> @@ -666,6 +666,7 @@
>  )
>  # append the last entry
> +        if curr_stream:
> stream.append(curr_stream)
> ```

Analysis: Both patches add the same guard (`if curr_stream`), but the pointwise-selected patch (idx=0) has an *indentation error*—the `stream.append` line is not nested under the new `if` block. This subtle formatting difference makes the patch syntactically different from the intended fix. Pairwise comparison helped the verifier notice this indentation discrepancy when directly contrasting the two candidates.

G.4 Example 4: `django__django-14017`: Pointwise Correct, Pairwise Wrong

> **Issue: Q(...) & Exists(...) raises a TypeError**
>
> `Exists(...) & Q(...)` works, but `Q(...) & Exists(...)` raises a `TypeError`. The `&` (and `|`) operators should be commutative on Q-Exists pairs, but they are not. The issue is in `Q._combine()` which checks `isinstance(other, Q)` and raises `TypeError` for non-Q operands like `Exists`.

Setup: 8 candidate patches generated, 7 non-empty.
Pairwise ranking: $[5, 4, 2, 0, 3, 6, 1] \quad \rightarrow$ selected **idx=5** ✗
Pointwise ranking: $[2, 3, 4, 5, 6, 1, 0] \quad \rightarrow$ selected **idx=2** ✓

> **Pointwise-Selected Patch (idx=2): Resolves the issue** ✓
>
> ```
> diff -git a/django/db/models/query_utils.py
> @@ -40,7 +40,10 @@ class Q(tree.Node):
>  def _combine(self, other, conn):
> -    if not isinstance(other, Q):
> +    from django.db.models.expressions import Expression
> ```

```
+    if isinstance(other, Expression):
+        other = Q(other)
+    elif not isinstance(other, Q):
raise TypeError(other)
```

Pairwise-Selected Patch (idx=5): Does not resolve the issue ✗

```
diff -git a/django/db/models/expressions.py
# Moves Q import to local scope + modifies __and__ in Combinable

diff -git a/django/db/models/query_utils.py
@@ -40,6 +41,8 @@ class Q(tree.Node):
def _combine(self, other, conn):
+    if isinstance(other, expressions.Expression) and getattr(other, "conditional", False):
+        other = Q(other)
if not isinstance(other, Q):
raise TypeError(other)
# Also modifies Q.deconstruct() for Expression children
```

Analysis: The correct patch (idx=2) is a minimal, focused fix: it wraps any `Expression` operand in `Q()` within `_combine`. The pairwise-selected patch (idx=5) attempts a more complex fix spanning two files and adding a `conditional` attribute check, but introduces regressions in `deconstruct()` handling. This example illustrates a failure mode of pairwise verification: a more complex-looking patch can appear "more thorough" in head-to-head comparison, even when the simpler patch is correct.

G.5 Example 5: `scikit-learn__scikit-learn-13439`: Both Correct, Different Indices

Issue: Pipeline should implement __len__

With the new indexing support, `pipe[:len(pipe)]` raises an error because `Pipeline` does not implement `__len__`.
```
from sklearn.pipeline import Pipeline
pipe = Pipeline([('anova', anova_filter), ('svc', clf)])
len(pipe) # TypeError:  object of type 'Pipeline' has no len()
```

Setup: 8 candidate patches generated, all non-empty.
Pairwise ranking: $[7, 4, 3, 1, 2, 5, 0, 6]$ → selected **idx=7** ✓
Pointwise ranking: $[1, 2, 3, 4, 5, 7, 0, 6]$ → selected **idx=1** ✓

Pairwise-Selected Patch (idx=7): Resolves the issue ✓

```
diff -git a/sklearn/pipeline.py
@@ -130,6 +130,12 @@ class Pipeline(_BaseComposition):
self.memory = memory
+  def __len__(self):
+    """Returns the length of the Pipeline"""
+    return len(self.steps)
```

Pointwise-Selected Patch (idx=1): Also resolves the issue ✓

```
diff -git a/sklearn/pipeline.py
@@ -27,6 +27,8 @@ class Pipeline(_BaseComposition):
+  def __len__(self):
+    return len(self.steps)
```

Analysis: Both patches correctly implement `__len__` by returning `len(self.steps)`. They differ only in placement within the class (after `__init__` vs. at the top of the class) and the presence of a docstring. This example demonstrates that when multiple correct solutions exist among the candidates, both verification methods may successfully identify a correct patch, even if they select different candidates.